Baryogenesis from Quantum Fluctuations during Inflation

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Introduction

Linking Baryogenesis and Inflation

- Relate observables such as $P_{\mathcal{R}}$, $n_{\rm s}$, r to baryogenesis
- For example hybrid inflation
  - Hybrid inflation and leptogenesis (Senoguz and Shafi 2004-2007): Inflaton decays directly into RH neutrinos through a non-renormalizable operator
    $\rightarrow$ Relations between reheat temperature, baryon asymmetry
  - F_D-inflation (Pilaftsis *et. al.* 2006): Inflaton couples at renormalizable level to RH neutrinos and is responsible for RH neutrino mass
    $\rightarrow$ Gravitiono problem is addressed and additional link to the scalar spectral index $n_{\rm s}$

Introduction

- In this talk, consider the possibility of linking the baryon asymmetry of the Universe (BAU) to inflationary observables without directly coupling the matter to the inflation sector
- BAU is created from vaccuum fluctuations during inflation
- Mechanism and calculation very analogous to case of scalar density perturbations
- This way, BAU appears a intial condition of the hot big bang scenario
- We find that the proposed mechanism is viable
- Requires a high energy scale of inflation, and can therefore be constrained by tighter bounds on r available in the near future

Lagrangian and Symmetries

- Scalar Lagrangian with non-standard kinetic and CP-violating mass term:

$$\mathcal{L}_\phi = \sqrt{-g}\left[\varrho g^{\mu\nu}(\partial_\mu\phi)\partial_\nu\phi^* - m^2\phi\phi^* + \left(\frac{\omega}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\mu^2}{2}\phi^2 + \text{c.c.}\right)\right]$$

- Coupling to fermions:

$$\mathcal{L}_\text{f} = \sqrt{-g}\left[\overline{\psi}\text{i}\partial\!\!\!/\psi + \overline{\chi}\text{i}\partial\!\!\!/\chi - f\left(\overline{\psi}\phi\chi + \text{h.c.}\right)\right]$$

- Symmetries of $\mathcal{L}_\text{f}$ (violated in $\mathcal{L}_\phi$).

$$\begin{aligned}\psi &\rightarrow \text{e}^{-\text{i}\beta}\psi\,,\\ \chi &\rightarrow \text{e}^{-\text{i}(\alpha+\beta)}\chi\,,\\ \phi &\rightarrow \text{e}^{\text{i}\alpha}\phi\end{aligned}$$

Currents

- Fermion current is conserved:

$$j_{\rm f}^{\mu} = \sqrt{-g}\left(-\frac{\beta}{\alpha}\bar{\psi}\Gamma^{\mu}\psi - \frac{\alpha+\beta}{\alpha}\bar{\chi}\Gamma^{\mu}\chi\right)\,, \qquad \Gamma^{\mu} = \frac{1}{a}\gamma^{\mu}\,, \quad D_{\mu}j_{\rm f}^{\mu} = 0$$

- Scalar current non-conserved:

$$j_{\phi}^{\mu} = \sqrt{g}g^{\mu\nu}\left\{{\rm i}\varrho\left[(\partial_{\nu}\phi^{*})\phi - \phi^{*}(\partial_{\nu}\phi)\right] + \left[{\rm i}\omega(\partial_{\nu}\phi)\phi + {\rm c.c.}\right]\right\}$$

$$D_{\mu}j_{\phi}^{\mu} = \sqrt{-g}\left\{{\rm i}\omega g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi - {\rm i}\mu^{2}\phi^{2} + {\rm h.c.}\right\}$$

Parametrization of de Sitter background

Conformal coordinates: $g_{\mu\nu} = a^{2}\eta_{\mu\nu}\,, \quad a = -\frac{1}{H\eta}\,, \quad \eta \in]-\infty;0[$

Comoving time: $dt = ad\eta\,, \quad a = {\rm e}^{Ht}$

Recipe

- Source CP-violation by the parameters ω and μ
- Choose the mass of the canonically normalized scalar degrees of freedom to be smaller than the Hubble rate H during inflation

Then, the same way scalar density perturbations are generated during inflation, also a charge density emerges.
This charge is conserved in the decay of ϕ to fermions and within the subsequent evolution of the Universe after the decays.

- Similar to the curvaton mechanism. Instead of curvature, we generate however baryons.

Real field decomposition

- introduce real degrees of freedom $\phi_\pm$ ...

$$\phi = \alpha\phi_+ + i\beta\phi_- \,, \quad \phi^\dagger = \alpha^*\phi_+ - i\beta^*\phi_-$$

- ... and impose the kinetic terms to be canonically normalized

$$\mathcal{L}_\phi = \sqrt{-g}\left[\frac{1}{2}\sum_\pm g^{\mu\nu}(\partial_\mu\phi_\pm)\partial_\nu\phi_\pm - \frac{1}{2}m_+^2\phi_+^2 - \frac{1}{2}m_-^2\phi_-^2 - m_{+-}^2\phi_+\phi_-\right]$$

Mass terms

- All CP violation now resides in the mass terms:

$$
\begin{aligned}
m_+^2 &= \frac{\varrho}{\varrho^2 - |\omega|^2}\left[m^2 + |\mu|^2 \cos(\theta_{\rm CP} + \theta_+ + \theta_-)\right] \\
&= \frac{\varrho}{\varrho^2 - |\omega|^2}\left[m^2 - |\mu|^2 \left(\frac{|\omega|}{\varrho}\cos(\theta_{\rm CP}) + \sqrt{1 - \frac{|\omega|^2}{\varrho^2}}\sin(\theta_{\rm CP})\right)\right], \\
m_-^2 &= \frac{\varrho}{\varrho^2 - |\omega|^2}\left[m^2 - |\mu|^2 \cos(\theta_{\rm CP} + \theta_+ - \theta_-)\right] \\
&= \frac{\varrho}{\varrho^2 - |\omega|^2}\left[m^2 - |\mu|^2 \left(\frac{|\omega|}{\varrho}\cos(\theta_{\rm CP}) - \sqrt{1 - \frac{|\omega|^2}{\varrho^2}}\sin(\theta_{\rm CP})\right)\right], \\
m_{+-}^2 &= \frac{\varrho}{\varrho^2 - |\omega|^2}\left[m^2 \sin(\theta_-) - |\mu|^2 \sin(\theta_{\rm CP} + \theta_+)\right] \\
&= \frac{\varrho}{\varrho^2 - |\omega|^2}\left[m^2 \frac{|\omega|}{\varrho} - |\mu|^2 \cos(\theta_{\rm CP})\right]
\end{aligned}
$$

- CP-angle:

$$
\theta_{\rm CP} = 2\theta_\mu - \theta_\omega\,, \quad \omega = |\omega| e^{i\theta_\omega}\,, \quad \mu = |\mu| e^{i\theta_\mu}
$$

Mixing degrees of freedom

- Reassemble scalar field

$$\Phi = \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}$$

- Real symmetric mass matrix:

$$M^2 \equiv \begin{pmatrix} m_+^2 & m_{+-}^2 \\ m_{+-}^2 & m_-^2 \end{pmatrix}$$

- Equations of motion take simple (and familiar) form:

$$\left(\partial_\eta^2 - \partial_i^2 + a^2 M^2 - \frac{a''}{a}\right)(a\Phi) = 0$$

Quantization

- Canonical commutation relations

$$\left[\hat{\phi}_a(\vec{x},\eta),\hat{\pi}_b(\vec{x}\,',\eta)\right]=\mathrm{i}\delta_{ab}\delta^3(\vec{x}-\vec{x}\,')\qquad (a,b=\pm,\ \hbar=1)$$

  Canonical momentum:

$$\hat{\pi}_a=\frac{\partial\mathcal{L}_\phi}{\partial\partial_\eta\phi_a}=a^2\hat{\phi}_a'\,,\qquad (a=\pm)$$

- Assemble components

$$\hat{\Phi}=\begin{pmatrix}\hat{\phi}_+\\ \hat{\phi}_-\end{pmatrix},\qquad \hat{\Pi}=\begin{pmatrix}\hat{\pi}_+\\ \hat{\pi}_-\end{pmatrix}$$

Quantization

- Mode expansion

$$\Phi(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left[\Phi_k(\eta) \cdot \hat{A}_{\vec{k}} + \Phi_k^*(\eta) \cdot \hat{A}^\dagger_{-\vec{k}}\right]$$

- Mode functions are matrix valued

$$\Phi_k = \begin{pmatrix} \phi_{k,++} & \phi_{k,+-} \\ \phi_{k,-+} & \phi_{k,--} \end{pmatrix}, \qquad \hat{A}_{\vec{k}} = \begin{pmatrix} \hat{a}_{\vec{k},+} \\ \hat{a}_{\vec{k},-} \end{pmatrix}.$$

Note: M^2 symmetric $\longrightarrow$ $\Phi_k(\eta)$ and $\Phi'_k(\eta)$ symmetric

$$\Phi_k^*(\eta) = \Phi_k^\dagger(\eta)$$

Mode solutions

- Fundamental solutions:

$$\Xi_k(\eta) = \frac{1}{a}\sqrt{-\frac{\pi\eta}{4}}\mathrm{e}^{\mathrm{i}(\pi/2)(\nu-1/2)}H_\nu^{(1)}(-k\eta)\,, \quad \Xi_k^\dagger(\eta) \qquad \left(\nu^2 = \frac{9}{4} - \frac{M^2}{H^2}\right)$$

- Matrix generalization of the standard Bunch-Davies (1978) vacuum solution.
- Mode functions are linear combinations in general

$$\Phi_k(\eta) = \gamma_1 \cdot \Xi_k(\eta) + \gamma_2 \cdot \Xi_k(\eta)^\dagger\,, \qquad \Phi^\dagger = \Xi_k^\dagger(\eta)\cdot\gamma_1 + \Xi_k(\eta)\cdot\gamma_2$$

- Wronskian required to satisfy commutation relations:

$$W[\Xi_k(\eta), \Xi_k^\dagger(\eta)] = \Xi_k(\eta)\cdot\Xi_k^{\dagger\prime}(\eta) - \Xi_k'(\eta)\cdot\Xi_k^\dagger(\eta) = \frac{\mathrm{i}}{a^2}$$

- Canonical commutation relations satisfied if

$$\Longrightarrow \gamma_1\cdot\gamma_1^\dagger - \gamma_2\cdot\gamma_2^\dagger = \mathbf{1}$$

Vacuum State

- For $\eta \to -\infty$:

$$a\,\Xi_k(\eta) \sim \mathrm{diag}(1,1)\frac{1}{\sqrt{2k}}\mathrm{e}^{-\mathrm{i}k\eta}$$

- To be associated with the annihilation operator when curvature becomes unimportant (large blue shift)
- Vacuum state corresponding to zero particles as $\eta \to -\infty$:

$$\gamma_1 = \mathrm{diag}(1,1)\,, \quad \gamma_2 = \mathrm{diag}(0,0)$$

Scalar Current and Charge

- Current operator:

$$\hat{j_\phi}^\mu = a^4 \left\{ \mathrm{i}\varrho \left[(\partial^\mu \hat{\phi}^\dagger)\hat{\phi} - \hat{\phi}^\dagger(\partial^\mu\hat{\phi}) \right] + \mathrm{i}\omega(\partial^\mu\hat{\phi})\hat{\phi} - \mathrm{i}\omega^*(\partial^\mu\hat{\phi}^\dagger)\hat{\phi}^\dagger \right\}$$
$$= a^4 \Big\{ -\frac{|\omega|}{\sqrt{\varrho^2 - |\omega|^2}} \left[\hat{\phi}_+ \partial^\mu \hat{\phi}_+ - \hat{\phi}_- \partial^\mu \hat{\phi}_- \right]$$
$$+ \frac{\varrho}{\sqrt{\varrho^2 - |\omega|^2}} \left[\hat{\phi}_+ \partial^\mu \hat{\phi}_- - \hat{\phi}_- \partial^\mu \hat{\phi}_+ \right] \Big\}$$

- Charge:

$$q_\phi = \frac{{j_\phi}^0}{a^3} = -\frac{1}{2}\frac{|\omega|}{\sqrt{\varrho^2 - |\omega|^2}} \partial_t \int \frac{d^3k}{(2\pi)^3} \left[|\Phi_k(\eta)|^2_{++} - |\Phi_k(\eta)|^2_{--} \right]$$
$$+ \frac{1}{2}\frac{\varrho}{\sqrt{\varrho^2 - |\omega|^2}} \partial_t \int \frac{d^3k}{(2\pi)^3} \left[|\Phi_k(\eta)|^2_{+-} - |\Phi_k(\eta)|^2_{-+} \right]$$

- Φ symmetric $\Longrightarrow$ 2nd line does not contribute

de Sitter propagator

- Mode sum:

$$i\Delta(x,x') = \langle 0|\Phi(x')\Phi^\dagger(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\Phi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}\Phi_k^*(\eta')e^{-i\mathbf{k}\cdot\mathbf{x}'}$$

  Chernikov-Tagirov (1968) solution :

$$i\Delta(x,x') \;=\; \frac{\Gamma(\frac{3}{2}+\nu)\Gamma(\frac{3}{2}-\nu)}{(4\pi)^2}H^2\;{}_2F_1\left(\frac{3}{2}+\nu,\frac{3}{2}-\nu;2;1-\frac{y}{4}\right)$$

- dS-invariant distance (ℓ is geodesic disance):

$$y = \frac{-(\eta-\eta')^2+\mathbf{x}^2}{\eta\eta'} = 4\sin^2\left(\frac{1}{2}H\ell\right)$$

- Expansion in y:

$$i\Delta(x,x') \;=\; \frac{H^2}{4\pi^2}\frac{1}{y} + \frac{H^2}{16\pi^2}\sum_{n=0}^{\infty}\frac{\Gamma\left(\frac{3}{2}+\nu+n\right)\Gamma\left(\frac{3}{2}-\nu+n\right)}{\Gamma\left(\frac{1}{2}+\nu\right)\Gamma(\frac{1}{2}-\nu)}\left(\frac{y}{4}\right)^n$$

$$\times\left[\log\frac{y}{4}+\psi\left(\frac{3}{2}+\nu+n\right)+\psi\left(\frac{3}{2}-\nu+n\right)-\psi\left(1+n\right)-\psi\left(2+n\right)\right]$$

Charge Asymmetry from Inflation

- Next, expand in M^2/H^2

$$\mathrm{i}\Delta(x,x') = \frac{H^2}{4\pi^2}\frac{1}{y} + \frac{3H^4}{8\pi^2 M^2} + \left(\frac{\log 2}{4\pi^2} - \frac{7}{24\pi^2} - \frac{\log y}{8\pi^2}\right) H^2 + \left(\frac{\log y}{16\pi^2} - \frac{\log 2}{8\pi^2} - \frac{1}{216\pi^2} + O(y)\right) M^2 + O\left(\frac{M^4}{H^2}\right)$$

- Note: $(\partial_t + \partial_{t'}) \log y|_{x=x'} = H$

$$q_\phi = \frac{1}{8\pi^2}|\omega\mu^2| H \sin\theta_{\mathrm{CP}} \frac{1}{\varrho^2 - |\omega|^2}$$

- Can be related to scale of inflation

Validity of Expansion

- Need to require $|M|^2 \lesssim H^2$
- Mass eigenvalues

$$M_\pm^2 = \frac{\varrho}{\varrho^2 - |\omega|^2}\left[m^2 - \frac{|\omega\mu^2|}{\varrho}\cos\theta_{\rm CP} \right. \\ \left. \pm\sqrt{|\mu|^4\left(1-\frac{|\omega|^2}{\varrho^2}\right) + m^4\frac{|\omega|^2}{\varrho^2} - 2\frac{|\omega\mu^2|}{\varrho^2}m^2\cos\theta_{\rm CP}}\right]$$

- Also estimate $|\omega\mu^2|/(\varrho^2 - |\omega|^2) \lesssim H^2$ (important to find bound on baryon assymmetry)
- $\Longrightarrow$ Cannot achieve resonance by setting $\varrho \sim |\omega|$

Present upper Bound on Scale of Inflation

- Hubble rate and potential

$$H^2 = \frac{8\pi}{3}\frac{V}{m_{\rm Pl}^2}$$

- Relation to tensor-to-scalar ratio

$$V \approx (3.3 \times 10^{16}{\rm GeV})^4 r$$

- WMAP3+SDSS (no running of $n_{\rm s}$): $r < 0.30$ at 95% c.l.
- $\Longrightarrow V^{1/4} < 2.4 \times 10^{16}{\rm GeV}$
- Future bounds: Planck $r < 0.1$, CLOVER $r < 0.01$
- For comparison: $V = \frac{1}{2}m^2\phi_{\rm I}^2$-inflation: $V^{1/4} \approx 2.0 \times 10^{16}{\rm GeV}$ at horizon exit; $V^{1/4} \approx 0.5 \times 10^{16}{\rm GeV}$ at the end of inflation

Lyth bound

- For 50 e-folds of Inflation (Lyth 1997)

$$\Delta\phi_{\rm I} \approx 11 m_{\rm Pl} r^{1/2}$$

- Models with large r require to trans-Planckian Inflaton VEVs
- These models are ill-motivated within supergravity

Baryon Asymmetry and Scale of Inflation

$$q_\phi = \frac{1}{8\pi^2}|\omega\mu^2| H \sin\theta_{\rm CP} \frac{1}{\varrho^2 - |\omega|^2}$$

- Conserved charge-to-entropy ratio after reheating:

$$\frac{q_\phi^{\rm R}}{s} = \left(\frac{a_{\rm end}}{a_{\rm R}}\right)^3 \frac{q_\phi}{s} = \frac{3}{4}\frac{T_{\rm R}}{V} q_\phi$$

- $T_{\rm R} = \left(\frac{30\varrho_{\rm E}}{\pi^2 g_*}\right)^{1/4}$; Instant reheating ($g_* = 106.75$): $T_{\rm R}/V^{1/4} = 0.41$
- Observed BAU (WMAP3): $\frac{n_B}{s} \simeq 8.7 \times 10^{-11}$
- For $q_\phi = \tau H^3$:

$$\frac{n_B}{s} \sim \frac{q_\phi^{\rm R}}{s} = 17.2 \times (8.7 \times 10^{-11})\tau \frac{T_{\rm R}}{0.41\, V^{1/4}} \left(\frac{V^{1/4}}{10^{16}{\rm GeV}}\right)^3$$

- Assuming instant reheating, the present 95% upper bound on r implies:

$$\tau > 4.3 \times 10^{-3}$$

Sketch of a possible connection to the Standard Model

- $\mathrm{U}(1)_{B-L}$-violating condensate X at energy scale Λ
- CP-violating parameters can be of the origin:

$$\omega = h_\omega X^{*2}\,, \qquad \mu^2 = h_\mu X^{*2}$$

- For example, assume couplings

$$\lambda X \phi^{*2}\phi + \lambda^* X^* \phi^2 \phi^*$$

- Self-energy for $(\ \phi \quad \phi^*\ )^T$

$$\mathrm{i}\Pi(p) = -\frac{\mathrm{i}}{8\pi^2}\begin{pmatrix} (\lambda X)^2 + (\lambda X)^{*2} & |\lambda X|^2 \\ |\lambda X|^2 & (\lambda X)^2 + (\lambda X)^{*2} \end{pmatrix}\left(\log\frac{\Lambda^2}{m^2} + \frac{p^2}{6m^2}\right)$$

(have expanded in $p^2 < 4m^2$ and $m^2 < \Lambda$.

- $\Pi(p^2) \longrightarrow \mu^2$-term; $\qquad \partial\Pi(p^2)/\partial p^2 \longrightarrow \omega$-term
- Decay to baryons by introducing a fermion $\widetilde{H}$ with same gauge quantum numbers as the Standard Model Higgs field, L is the left-handed lepton doublet

$$\phi L \widetilde{H} + \mathrm{h.c.}$$

Conclusions

- Baryogenesis from the amplification of vacuum fluctuations during inflation is possible.
- Large scale of inflation is required, close to present observational bounds.
- High reheat temperatures are required.
- Not a good scenario for local SUSY.
- However, a nice scenario in the case Planck or CLOVER discover tensor perturbations.