Connecting Leptogenesis to Low Energy LFV Processes in Models with Spontaneous CP Violation

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work done in collaboration with K.T. Mahanthappa, Phys. Rev. D71, 035001 (2005)
Phys. Rev. D75, 015001 (2007)

KICP Workshop on Baryogenesis Confronts Experiment, University of Chicago, November 7, 2007

Baryon Number Asymmetry in SM

CP violation in quark sector not sufficient to explain the observed matter -anti-matter asymmetry of the Universe

$$n_b/s = (0.87 \pm 0.04) \times 10^{-10}$$

- CP phase in CKM matrix:

$$B \simeq \frac{\alpha_w^4 T^3}{s} \delta_{CP} \simeq 10^{-8} \delta_{CP} \qquad \delta_{CP} \simeq \frac{A_{CP}}{T_C^{12}} \simeq 10^{-20}$$

$$A_{CP} = (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2) \cdot J$$

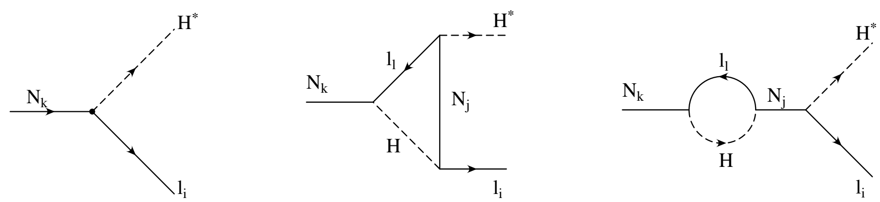
- effects of CP violation suppressed by small quark mixing

$\longrightarrow$ $B \sim 10^{-28}$

too small to account for the observed $B \sim 10^{-10}$

Leptogenesis

- non-zero neutrino masses: additional CPV sources from lepton sector
- primordial lepton number asymmetry due to out-of-equilibrium decays of RH neutrinos



$$\epsilon_1 \simeq -\frac{3}{8\pi}\frac{1}{(h_\nu h_\nu^\dagger)_{11}}\sum_{i=2,3}\mathrm{Im}\left\{(h_\nu h_\nu^\dagger)^2_{1i}\right\}\frac{M_1}{M_i}$$

- sphaleron effects: $\Delta L \rightarrow \Delta B$

Connection to Low Energy Observables

- Lagrangian at high energy (in the presence of RH neutrinos)

$$\mathcal{L} = \overline{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \overline{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + \overline{N}_{R_i} i\gamma^\mu \partial_\mu N_{R_i}$$
$$+ f_{ij} \overline{e}_{R_i} \ell_{L_j} H^\dagger + h_{ij} \overline{N}_{R_i} \ell_{L_j} H - \frac{1}{2} M_{ij} N_{R_i} N_{R_j} + h.c.$$

in f_{ij} and M_{ij} diagonal basis →

h_{ij} general complex matrix: { 9-3 = 6 mixing angles; 9-3 = 6 physical phases }

- Low energy effective Lagrangian (after integrating out RH neutrinos)

$$\mathcal{L}_{eff} = \overline{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \overline{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + f_{ii} \overline{e}_{R_i} \ell_{L_i} H^\dagger + \frac{1}{2} \sum_k h^T_{ik} h_{kj} \ell_{L_i} \ell_{L_j} \frac{H^2}{M_k} + h.c.$$

in f_{ij} diagonal basis →

h_{ij} symmetric complex matrix: { 6-3 = 3 mixing angles; 6-3 = 3 physical phases }

- high energy → low energy:

numbers of mixing angles and CP phases reduced by half

Connection to Low Energy Observables

- diagonal basis for charged lepton and RH neutrino mass matrices
- neutrino Yukawa interactions $h = V_R^{\nu\,\dagger} \mathrm{diag}(h_1, h_2, h_3) V_L^{\nu}$
- CP asymmetry parametrized by (orthogonal parametrization)

(Casas & Ibarra, 2001)

$m = \mathrm{diag}(m_1, m_2, m_3)$ (light neutrino masses)

$M = \mathrm{diag}(M_1, M_2, M_3)$ (RH neutrino masses)

$R = vM^{-1/2} h U m^{-1/2}$ R: phases in RH sector

$$h h^\dagger v^2 = V_R^{\nu\,\dagger} \mathrm{diag}(h_1^2, h_2^2, h_3^2) V_R^{\nu} v^2 = M^{1/2} R m R^\dagger M^{1/2}$$

- lepton number asymmetry (in one-flavor approximation)

$$\epsilon_1 = \sum_{\alpha=e,\mu,\tau} \epsilon^{\alpha\alpha} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_\rho m_\rho^2 R_{1\rho}^2\right)}{\sum_\beta m_\beta |R_{1\beta}|^2}$$

Connection to Low Energy Observables

absence of low energy leptonic CPV (neutrino oscillation, neutrinoless double beta decay)

leptogenesis $\neq 0$

- Flavor matters?

  leptogenesis at T ~ $M_1 < 10^{12}$ GeV:

  three flavors distinguishable (different T_{eq})

  non-universal wash-out factors

- asymmetry associated with each flavor

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U^*_{\alpha\beta} U_{\alpha\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta |R_{1\beta}|^2}$$

leptogenesis $\neq 0$

low energy CPV $\neq 0$

Sources of CP Violation

- Manifestations of CP violation
  - weak scale CPV (kaon, B-meson, neutrino oscillation, ...)
  - cosmological BAU
  - strong CP problem

  ⇒ can they come from a common origin??

- Explicit CP violation
  - complex Yukawa couplings
- Spontaneous CP violation
  - complex VEV
    - domain walls?
    - sufficiently large phases?

Minimal Left-Right Model

Pati, Salam, 1974;
Mohapatra, Pati, 1975;
Senjanovic, Mohapatra, 1975

- gauge symmetry: $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$

  $\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \qquad Q = T_{3,L} + T_{3,R} + \frac{1}{2}(B-L)$

- particle content:
  - fermions:

$$Q_{i,L} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,L} \sim (1/2,\ 0,\ 1/3) \qquad Q_{i,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,R} \sim (0,\ 1/2,\ 1/3)$$

$$L_{i,L} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,L} \sim (1/2,\ 0,\ -1) \qquad L_{i,R} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,R} \sim (0,\ 1/2,\ -1)$$

  - Higgs sector:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1/2,\ 1/2,\ 0)$$

$$\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta_L^+ & \Delta_L^{++} \\ \Delta_L^0 & -\frac{1}{\sqrt{2}}\Delta_L^+ \end{pmatrix} \sim (1,\ 0,\ +2) \qquad \Delta_R = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta_R^+ & \Delta_R^{++} \\ \Delta_R^0 & -\frac{1}{\sqrt{2}}\Delta_R^+ \end{pmatrix} \sim (0,\ 1,\ +2)$$

- Under P:

$$\psi_L \leftrightarrow \psi_R, \quad \Delta_L \leftrightarrow \Delta_R, \quad \Phi \leftrightarrow \Phi^+$$

Minimal Left-Right Model

- In general,

$$\langle\Phi\rangle = \begin{pmatrix} \kappa e^{i\alpha_\kappa} & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}, \quad \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \quad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\alpha_R} & 0 \end{pmatrix}$$

- To get realistic SM gauge boson masses:

$$\kappa^2 + \kappa'^2 \cong 2m_W^2 / g^2 \cong (174 GeV)^2$$

- two triplet VEV's are related

$$v_L = \beta \frac{\kappa^2}{v_R}$$

so υ_L is seesaw suppressed

Small neutrino masses: $\upsilon_R \sim 10^{15}$ GeV; $\upsilon_L \sim 0.01$ eV

⇒ precision EW constraints OK

Two Intrinsic CP Phases

- The Lagrangian is invariant under two unitary transformations

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix}, \quad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix}$$

- under U_L and U_R:

$$\psi_L \to U_L \psi_L, \quad \psi_R \to U_R \psi_R \qquad \Phi \to U_R \Phi U_L^+, \quad \Delta_L \to U_L^* \Delta_L U_L^+, \quad \Delta_R \to U_R^* \Delta_R U_R^+$$

$$\kappa \to \kappa e^{-i(\gamma_L - \gamma_R)}, \quad \kappa' \to \kappa' e^{i(\gamma_L - \gamma_R)}$$

$$v_L \to v_L e^{-2i\gamma_L}, \qquad v_R \to v_R e^{-2i\gamma_R}$$

- rotate away two of the four CP phases: only two physical phases remain

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

- scalar potential

Yukawa Interactions

- quarks: $-L_q = \overline{Q}_{i,R}(F_{ij}\Phi + G_{ij}\overline{\Phi})Q_{j,L} + h.c.$ where $\overline{\Phi} = \tau_2\Phi^*\tau_2$
- mass matrices $M_u = F_{ij}\kappa + G_{ij}\kappa' e^{-i\alpha_{\kappa'}}, \quad M_d = F_{ij}\kappa' e^{i\alpha_{\kappa'}} + G_{ij}\kappa$
  - SCPV ⇒ all Yukawa coupling constants real
  - $\alpha_{\kappa'}$ responsible for all CPV in quark sector
  - to suppress FCNC

⇒ large hierarchy between two doublet VEV's

$$\kappa/\kappa' \cong m_t/m_b >> 1$$

Yukawa Interactions

- lepton sector

$$-L_l = \overline{L}_{i,R}(P_{ij}\Phi + R_{ij}\overline{\Phi})L_{j,L} + f_{ij}(L^T_{i,L}\Delta_L L_{j,L} + L^T_{i,R}\Delta_R L_{j,R}) + h.c.$$

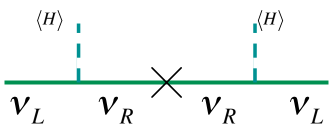
- mass matrices

$$M_e = P_{ij}\kappa' e^{i\alpha_{\kappa'}} + R_{ij}\kappa$$

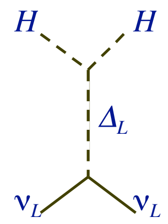
$$M_\nu^{Dirac} = P_{ij}\kappa + R_{ij}\kappa' e^{-i\alpha_{\kappa'}}, \quad M_\nu^{LL} = f_{ij} v_L e^{i\alpha_L}, \quad M_\nu^{RR} = f_{ij} v_R,$$

$$\begin{pmatrix} M_{LL} & M_{LR}^T \\ M_{LR} & M_{RR} \end{pmatrix}$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{\upsilon_R} SU(2)_L \times U(1)_Y \Rightarrow \begin{pmatrix} 0 & 0 \\ 0 & f\upsilon_R \end{pmatrix}$$



$$\xrightarrow{\upsilon_{ew}} U(1)_{EM} \Rightarrow \begin{pmatrix} f\upsilon_L & h\upsilon_{ew} \\ h\upsilon_{ew} & f\upsilon_R \end{pmatrix}$$



Yukawa Interactions

- lepton sector

$$M_e = P_{ij}\kappa' e^{i\alpha_{\kappa'}} + R_{ij}\kappa$$

$$M_\nu^{Dirac} = P_{ij}\kappa + R_{ij}\kappa' e^{-i\alpha_{\kappa'}}, \quad M_\nu^{LL} = f_{ij} v_L e^{i\alpha_L}, \quad M_\nu^{RR} = f_{ij} v_R,$$

$$M_\nu^{I} = (M_\nu^{Dirac})^T (M_\nu^{RR})^{-1} (M_\nu^{Dirac}) = (\kappa P + \kappa' e^{-i\alpha_{\kappa'}} R)^T (\upsilon_R f)^{-1} (\kappa P + \kappa' e^{-i\alpha_{\kappa'}} R) \approx \frac{\upsilon_L}{\beta} P^T f^{-1} P$$

$$M_\nu^{II} = \upsilon_L e^{i\alpha_L} f$$

$$M_\nu^{eff} = M_\nu^{II} - M_\nu^{I} \approx (f e^{i\alpha_L} - \frac{1}{\beta} P^T f^{-1} P)\upsilon_L$$

CPV in quark sector ↔ CPV in lepton sector

- through phase $\alpha_{\kappa'}$
- appear at sub-leading $O(\kappa'/\kappa)$
- weak connection

Leptonic CPV Processes

- MNS matrix

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha_{21}/2} & \\ & & e^{i\alpha_{31}/2} \end{pmatrix}$$

- three low energy phases δ, α_{21}, α_{31}: functions of α_L
- neutrino oscillations

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}) \sin^2(\Delta m^2_{ij} \tfrac{L}{4E}) + 2\sum_{i>j} J_{CP} \sin^2(\Delta m^2_{ij} \tfrac{L}{4E})$$

$$J_{CP} = -\frac{\mathrm{Im}(H_{12}H_{23}H_{31})}{\Delta m^2_{21}\Delta m^2_{32}\Delta m^2_{31}} \propto \sin\alpha_L, \qquad H \equiv M_\nu^{eff}(M_\nu^{eff})^+$$

- neutrinoless double beta decay

$$\left|\langle m_{ee}\rangle\right|^2 = m_1^2|U_{e1}|^4 + m_2^2|U_{e2}|^4 + m_3^2|U_{e3}|^4 + 2m_1m_2|U_{e1}|^2|U_{e2}|^2\cos\alpha_{21}$$
$$+ 2m_1m_3|U_{e1}|^2|U_{e3}|^2\cos\alpha_{31} + 2m_2m_3|U_{e2}|^2|U_{e3}|^2\cos(\alpha_{31} - \alpha_{21})$$

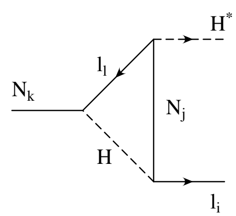
Leptogenesis in LR Model

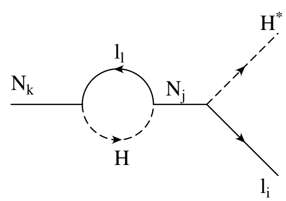
- two ways to generate lepton number asymmetry
  - decays of N_1: $N_1 \to \ell + H^*,$ $$\varepsilon = \frac{\Gamma(N_1 \to \ell + H^*) - \Gamma(N_1 \to \bar{\ell} + H)}{\Gamma(N_1 \to \ell + H^*) + \Gamma(N_1 \to \bar{\ell} + H)}$$
  - decays of Δ_L: $\Delta_L^* \to \ell + \ell,$ $$\varepsilon = \frac{\Gamma(\Delta_L^* \to \ell + \ell) - \Gamma(\Delta_L \to \bar{\ell} + \bar{\ell})}{\Gamma(\Delta_L^* \to \ell + \ell) + \Gamma(\Delta_L \to \bar{\ell} + \bar{\ell})}$$

- naturally $M_{\Delta_L} > M_1 \Rightarrow N_1$ decays dominate

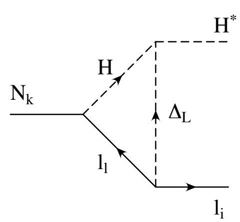
- contributions to the asymmetry

$$\mathrm{M}_D = O_R M_D$$





$$\varepsilon = \frac{3}{16\pi}\left(\frac{M_1}{\upsilon^2}\right)\frac{\mathrm{Im}(\mathrm{M}_D (M_\nu^I)^* \mathrm{M}_D^T)_{11}}{(\mathrm{M}_D \mathrm{M}_D^\dagger)_{11}} = 0$$



$$\varepsilon = \frac{3}{16\pi}\left(\frac{M_1}{\upsilon^2}\right)\frac{\mathrm{Im}(\mathrm{M}_D (M_\nu^{II})^* \mathrm{M}_D^T)_{11}}{(\mathrm{M}_D \mathrm{M}_D^\dagger)_{11}} \propto \sin\alpha_L$$

Independent of the choice of unitary transformation $U_{L,R}$

Leptogenesis in LR Model

- out-of-equilibrium condition

$$\frac{\Gamma}{H\,|_{T=M_1}} = \frac{M_{Pl}}{(1.7)(32\pi)\sqrt{g_*}\upsilon^2} \cdot \frac{\left(\mathrm{M}_D\mathrm{M}_D^+\right)_{11}}{M_1} \approx \frac{1}{(0.01eV)} \cdot \frac{\left(\mathrm{M}_D\mathrm{M}_D^+\right)_{11}}{M_1} < 1$$

➡ M_1 cannot be to light: typically requires $M_1 > 2 \times 10^9$ GeV

➡ RH neutrino masses cannot too hierarchical

$\Delta L \rightarrow \Delta B$:

observed $n_b / s \sim 10^{-10} \quad \Rightarrow \quad \varepsilon \sim 10^{-8}$

Specific Flavor Ansatz for Bi-large Mixing

$$M_\nu^{eff} = (f e^{i\alpha_L} - \frac{1}{\beta} P^T f^{-1} P)\upsilon_L$$

- assume Dirac neutrino mass matrix ~ up quark mass matrix

$$P \propto \begin{pmatrix} m_u/m_t & & \\ & m_c/m_t & \\ & & 1 \end{pmatrix}$$

- bi-large neutrino mixing can be accommodated with

$$f_{ij} = \begin{pmatrix} t^2 & t & -t \\ t & 1 & 1 \\ -t & 1 & 1 \end{pmatrix} \qquad \frac{1}{\beta} P^T f^{-1} P = s \begin{pmatrix} 0 & \frac{1}{t}\frac{m_u m_c}{m_t^2} & -\frac{1}{t}\frac{m_u}{m_t} \\ \frac{1}{t}\frac{m_u m_c}{m_t^2} & 0 & \frac{m_c}{m_t} \\ -\frac{1}{t}\frac{m_u}{m_t} & \frac{m_c}{m_t} & 0 \end{pmatrix}$$

  - may arise from U(1) horizontal symmetry
  - deviation from maximal atmospheric mixing negligible

- Leptonic CPV: $$J_{CP} = -\frac{2st^2(1-t^2)\upsilon_L^6}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}\left(\frac{m_u}{m_t}\right)\sin\alpha_L$$

Results

- 3 model parameters: t, s, α_L
- experimental values for oscillation parameters

$$post-Neutrino2004:(1\sigma)$$

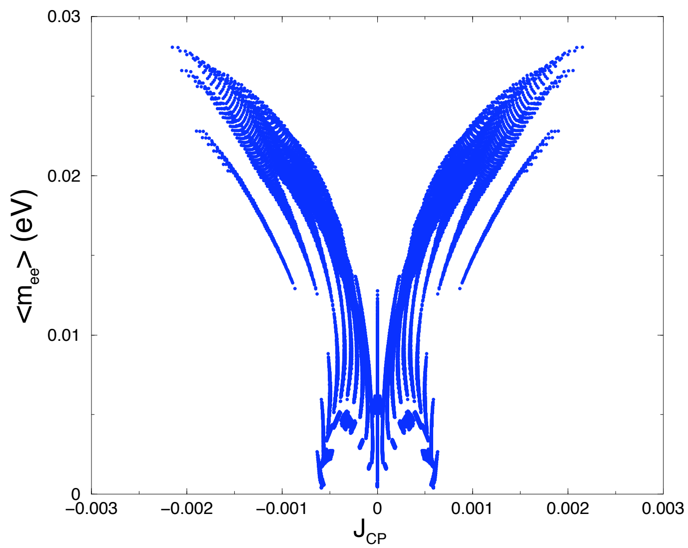
$$\Delta m^2_{atm} = (1.9-3.0)\times 10^{-3} eV^2,$$

$$\Delta m^2_{sol} = (7.9-8.5)\times 10^{-5} eV^2$$

$$\sin^2 2\theta_{atm} > 0.9$$

$$\tan^2\theta_{sol} = (0.35-0.44)$$

Results

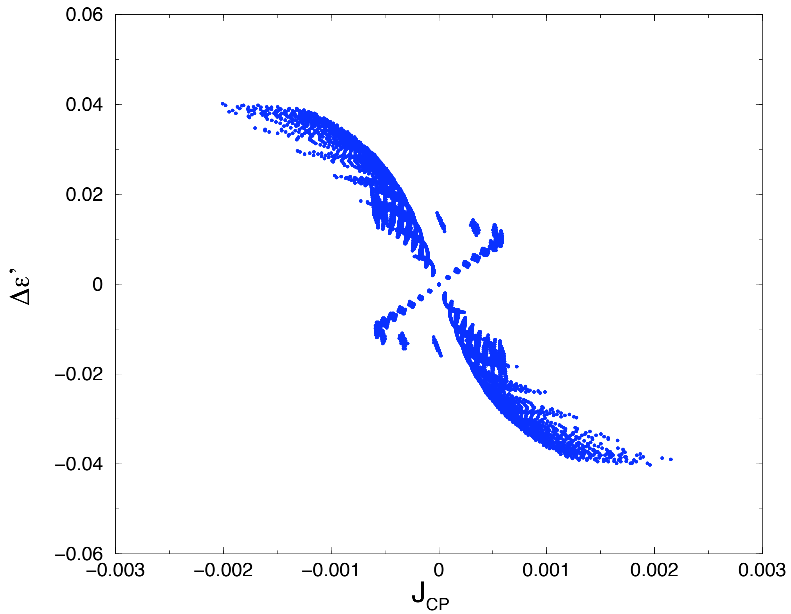


- predict small θ_{13}
- in large J_{cp} regime:

  strong correlation between J_{cp} and $\langle m_{ee} \rangle$
- J_{cp}: $(0 - 10^{-3})$
- $\langle m_{ee} \rangle$: $(10^{-4} \sim 10^{-2})$ eV; current limit ~ 0.1 eV

M.-C.C & Mahanthappa, Phys. Rev. D71, 035001 (2005)

Results



- symmetry between 2nd & 4th quadrants
- in large J_{cp} regime:
  strong correlation between J_{cp} and $\Delta\epsilon'$
- total amount of lepton number asymmetry

$$\varepsilon = 10^{-2} \times \Delta\varepsilon' < (10^{-4} - 10^{-5})$$

- no wash-out

$$\frac{\Gamma}{H|_{T=M_1}} \approx \frac{1}{(0.01eV)} \cdot \frac{(M_D M_D^+)_{11}}{M_1} < 1$$

$$\frac{(M_D M_D^+)_{11}}{M_1} \sim \left(\frac{m_c}{m_t}\right)^2 \upsilon_L = 10^{-7} eV!!!$$

$$\upsilon_R \sim (10^{12-13}) GeV, \;\; M_1 \sim 0.1 \upsilon_R$$

observed BAU ⇒ $J_{cp} \sim 10^{-5}$

Flavor Ansatz II

- assume Dirac neutrino mass matrix ~ up quark mass matrix

$$P \propto \begin{pmatrix} m_u/m_t & & \\ & m_c/m_t & \\ & & 1 \end{pmatrix}$$

- bi-large neutrino mixing can be accommodated with

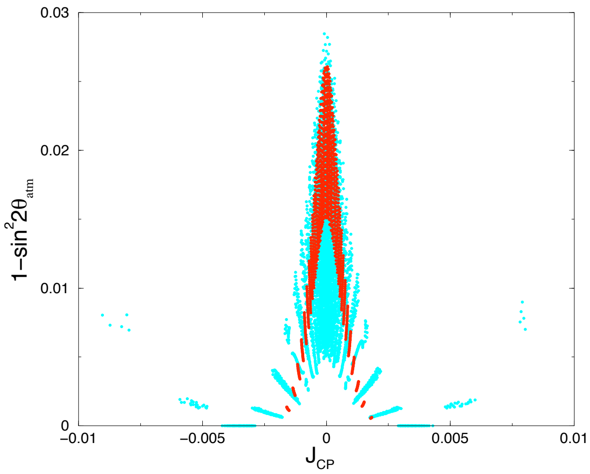
$$f_{ij} = \begin{pmatrix} A & B & -B \\ B & D+\frac{A}{2} & D-\frac{A}{2} \\ -B & D-\frac{A}{2} & D+\frac{A}{2} \end{pmatrix} \qquad A = \tfrac{1}{2}(f_1^0 + f_2^0),\ B = \tfrac{1}{2\sqrt{2}}(f_2^0 - f_1^0),\ D = \tfrac{1}{2} f_3^0$$

$$P^T f^{-1} P \sim \begin{pmatrix} & & \\ & & \\ & & x \end{pmatrix}, \quad x = s\left(\frac{1}{f_1^0} + \frac{1}{f_2^0} + \frac{1}{f_3^0}\right) \qquad M_\nu^{eff} = \left(f e^{i\alpha_L} - \frac{1}{\beta} P^T f^{-1} P\right)\upsilon_L$$

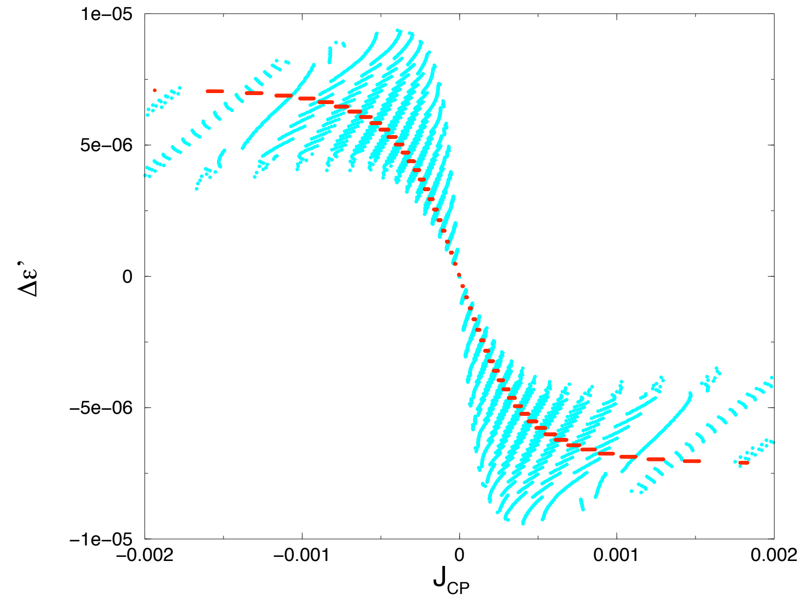
  - may arise from (L_e - L_μ - L_τ) horizontal symmetry
  - inverted hierarchy
  - deviation from maximal atmospheric mixing sizable

- Leptonic CPV: $$J_{CP} = -\frac{(f_2^0 - f_1^0)^3 (f_3^0 - f_2^0)(f_3^0 - f_1^0)\upsilon_L^6}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} x \sin\alpha_L$$

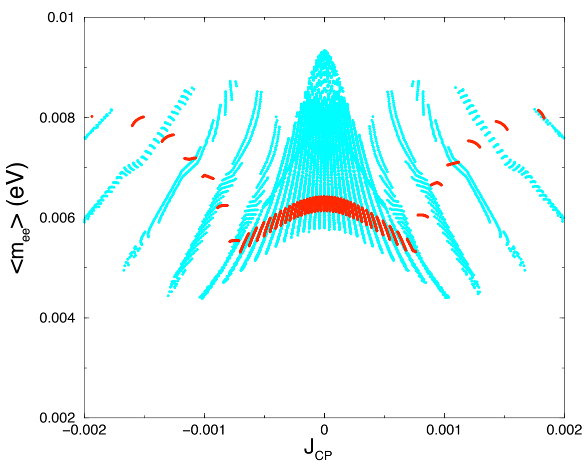
Results



0.03
0.02
0.01
0
1−sin²2θ_atm
−0.01
−0.005
0
0.005
0.01
J_CP



1e−05
5e−06
0
−5e−06
−1e−05
Δε'
−0.002
−0.001
0
0.001
0.002
J_CP



0.01
0.008
0.006
0.004
0.002
<m_ee> (eV)
−0.002
−0.001
0
0.001
0.002
J_CP

Scalar Potential

- scalar potential $V = V_\Phi + V_\Delta + V_{\Phi\Delta}$

$$
\begin{aligned}
V_\Phi &= -\mu^2 \mathrm{Tr}(\Phi^\dagger\Phi) - \mu_2^2\big[\mathrm{Tr}(\tilde\Phi\Phi^\dagger) + \mathrm{Tr}(\tilde\Phi^\dagger\Phi)\big] \\
&\quad +\lambda_1\big[\mathrm{Tr}(\Phi\Phi^\dagger)\big]^2 + \lambda_2\Big[\big[\mathrm{Tr}(\tilde\Phi\Phi^\dagger)\big]^2 + \mathrm{Tr}(\tilde\Phi^\dagger\Phi)\big]^2\Big] \\
&\quad +\lambda_3\big[\mathrm{Tr}(\tilde\Phi\Phi^\dagger)\mathrm{Tr}(\tilde\Phi^\dagger\Phi)\big] + \lambda_4\Big[\mathrm{Tr}(\Phi\Phi^\dagger)\big[\mathrm{Tr}(\tilde\Phi\Phi^\dagger) + \mathrm{Tr}(\tilde\Phi^\dagger\Phi)\big]\Big] \\
V_\Delta &= -\mu_3^2\big[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \Delta_R\Delta_R^\dagger)\big] + \rho_1\Big[\big[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\big]^2 + \big[\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\big]^2\Big] \\
&\quad +\rho_2\Big[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger)\Big] \\
&\quad +\rho_3\big[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\big] + \rho_4\Big[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger) + \mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R)\Big] \\
V_{\Phi\Delta} &= \alpha_1\Big[\mathrm{Tr}(\Phi\Phi^\dagger)\big[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)\big]\Big] \\
&\quad +\alpha_2\big[\mathrm{Tr}(\Phi\tilde\Phi^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\Phi^\dagger\tilde\Phi)\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\big]\Big] \\
&\quad +\alpha_2^*\big[\mathrm{Tr}(\Phi^\dagger\tilde\Phi)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\tilde\Phi^\dagger\Phi)\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\big]\Big] \\
&\quad +\alpha_3\big[\mathrm{Tr}(\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger)\big] + \beta_1\big[\mathrm{Tr}(\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger)\big] \\
&\quad +\beta_2\big[\mathrm{Tr}(\tilde\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\tilde\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger)\big] + \beta_3\big[\mathrm{Tr}(\Phi\Delta_R\tilde\Phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Delta_L\tilde\Phi\Delta_R^\dagger)\big]
\end{aligned}
$$

Scalar Potential

- minimization conditions

$$
\begin{aligned}
(2\rho_1 - \rho_3) v_R v_L &= \beta_1 \kappa\kappa' \cos(\alpha_L - \alpha_{\kappa'}) + \beta_2 \kappa^2 \cos\alpha_L + \beta_3 \kappa'^2 \cos(\alpha_L - 2\alpha_{\kappa'}) \,, \\
0 &= \beta_1 \kappa\kappa' \sin(\alpha_L - \alpha_{\kappa'}) + \beta_2 \kappa^2 \sin\alpha_L + \beta_3 \kappa'^2 \sin(\alpha_L - 2\alpha_{\kappa'})
\end{aligned}
$$

$$
\begin{aligned}
0 = v_R v_L \Big[2\kappa\kappa'(\beta_2 + \beta_3) \sin(\alpha_L - \alpha_{\kappa'}) + \beta_1 \left[\kappa^2 \sin\alpha_L + \kappa'^2 \sin(\alpha_L - 2\alpha_{\kappa'})\right]\Big] \\
+ \kappa\kappa' \sin\alpha_{\kappa'} \left[\alpha_3 (v_L^2 + v_R^2) + (4\lambda_3 - 8\lambda_2)(\kappa^2 - \kappa'^2)\right]
\end{aligned}
$$

- CP phase in quark sector: 3rd condition

$$\sin\alpha_{\kappa'} \sim (\beta/\alpha_3)(v_L/v_R)$$

⇒ suppressed by high υ_R

- CP phase in lepton sector: α_L

⇒ tuning of $O(\kappa'/\kappa) \sim (m_b/m_t) \sim 0.01$ (from FCNC constraints),

$\alpha_L \sim O(1)$ can be generated

Low Seesaw Scale

M.-C.C & Mahanthappa, Phys. Rev. D75, 015001 (2007)

- with an addition $U(1)_s$ symmetry: $SU(2)_R$ breaking scale can be significantly lower
  - $U(1)_s$ broken by <S> ; Q(S) = +1
  - $U(1)_s$ forbids dim-4 operators
  - mass terms arise only through operators with higher dimensionality $\omega = \frac{\langle s \rangle}{M_S}$ < 1

$$M_{\nu_D} = P_{ij}\kappa\omega^{|Q(L_L)-Q(L_R)-Q(\Phi)|} + R_{ij}\kappa' e^{-i\alpha_{\kappa'}}\omega^{|Q(L_L)-Q(L_R)+Q(\Phi)|}$$

$$M_e = P_{ij}\kappa' e^{i\alpha_{\kappa'}}\omega^{|Q(L_L)-Q(L_R)-Q(\Phi)|} + R_{ij}\kappa\omega^{|Q(L_L)-Q(L_R)+Q(\Phi)|}$$

$$M_\nu^{RR} = f_{ij} v_R \omega^{|Q(\Delta_R)+2Q(L_R)|}, \quad M_\nu^{LL} = f_{ij} v_L e^{i\alpha_L}\omega^{|Q(\Delta_L)+2Q(L_L)|}$$

Low Seesaw Scale

M.-C.C & Mahanthappa, Phys. Rev. D75, 015001 (2007)

- induced triplet VEV

$$v_L \simeq \beta \frac{\kappa^2}{v_R} ,$$

$$\beta \sim \beta' \cdot \text{Max}\{\omega^{|Q(\Delta_R)-Q(\Delta_L)-2Q(\Phi)|},\ r\omega^{|Q(\Delta_R)-Q(\Delta_L)|},\ r^2\omega^{|Q(\Delta_R)-Q(\Delta_L)+2Q(\Phi)|}\} ,$$

- U(1) charge assignment consistent with LR symmetry:

$$Q(\Phi) = -Q(\tilde{\Phi}) = 2 ,\ Q(\Delta_L) = -Q(\Delta_R) = 4 ,\ Q(L_L) = -Q(L_R) = -2 ,$$

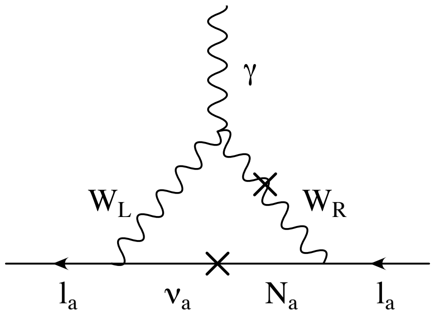
$$\begin{aligned} v_L v_R &\simeq \beta' \kappa^2 \omega^8 , \\ M_e &\simeq R_{ij} \kappa \omega^2 + \mathcal{O}(\omega^6) , \\ M_{\nu_D} &\simeq R_{ij} \kappa' e^{-i\alpha_{\kappa'}} \omega^4 + \mathcal{O}(\omega^6) , \\ M_\nu^{LL} &= f_{ij} v_L e^{i\alpha_L} , \quad M_\nu^{RR} = f_{ij} v_R \end{aligned}$$

$$\begin{aligned} M_\nu^{eff} &= M_\nu^{LL} - M_{\nu_D} (M_\nu^{RR})^{-1} M_{\nu_D}^T \\ &= v_L [f_{ij} e^{i\alpha_L} - s R_{ij} f_{ij}^{-1} R_{ij}^T e^{-2i\alpha_{\kappa'}}] \end{aligned}$$

with $\omega \sim 0.1$, $\upsilon_L \sim$ (0.01-0.1) eV $\Rightarrow$ $\upsilon_R \sim 10^6$ GeV

'quark' phase $\alpha_{\kappa'}$ now affect leptonic CPV at dominant order

Electron EDM



mixing between
W_L and W_R

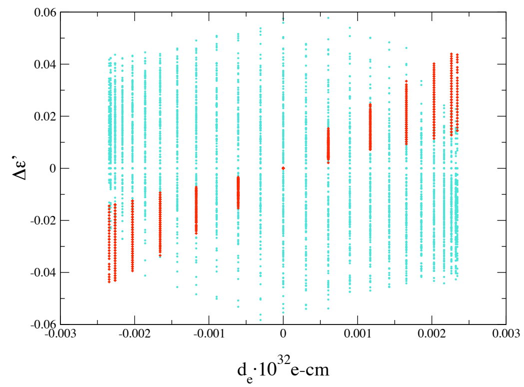
$$d_e \simeq -\frac{e\alpha}{4\pi M_W^2}\frac{\kappa\kappa'}{v_R^2 - v_L^2}\mathrm{Im}(e^{-i\alpha_{\kappa'}}M_D)_{ee}$$

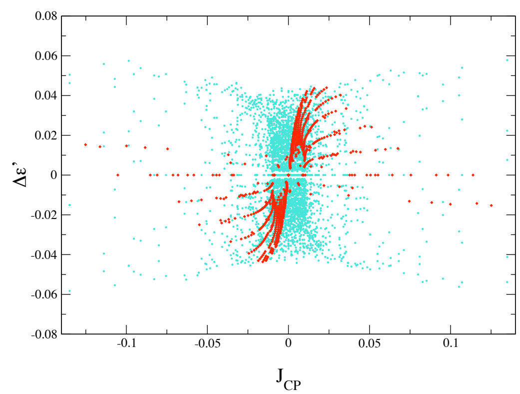
$$\simeq 10^{-19} \times r\left(\frac{GeV}{v_R}\right)^2\left(\frac{|(M_{\nu_D})_{ee}|}{MeV}\right)\sin(2\alpha_{\kappa'}) \quad \text{e-cm}$$

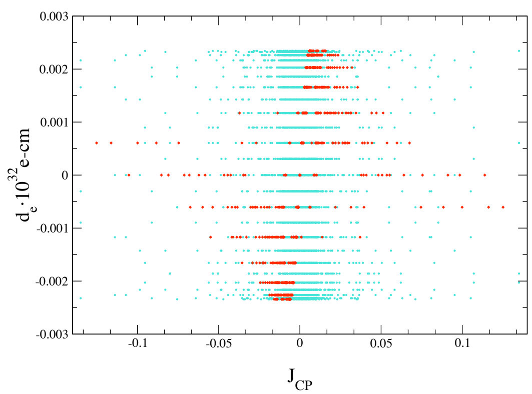
$v_R \sim 10^6$ GeV $\Rightarrow$ $d_e \sim 10^{-32}$ e-cm can be obtained
(within reach of next generation of experiments)
current limit: $d_e < 1.67\times10^{-27}$ e-cm at 90% CF

Results

$$J_{CP} \propto \sin(\alpha_L + \alpha_{\kappa'}) \qquad \epsilon^{\Delta_L} \propto M_{R_1} \sin(\alpha_L + 2\alpha_{\kappa'})$$







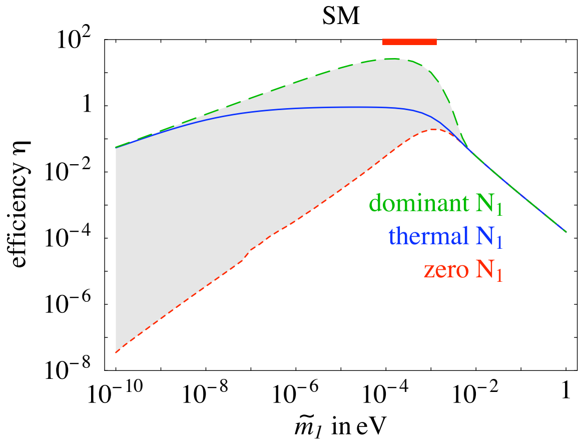
Results

$$\epsilon = \epsilon^{\Delta_L} \sim 10^{-8} \Delta\epsilon' \sim O(10^{-9})$$

$$n_b/s = (0.87 \pm 0.04) \times 10^{-10},$$

$$\frac{n_b}{s} \simeq -\frac{24 + 4n_H}{66 + 13n_H}\, \epsilon\, \eta\, Y^{\rm eq}_{N_1}(T \gg M_1)\,, \qquad n_b/s \simeq -1.38 \times 10^{-3}\, \epsilon\, \eta.$$

sufficient BAU can be obtained
with efficiency factor 10-100



Giudice, Notari, Raidal, Riotto & Strumia,
Nucl. Phys. B685, 89 (2004)

TeV Scale Seesaw with $U(1)_{NA}$

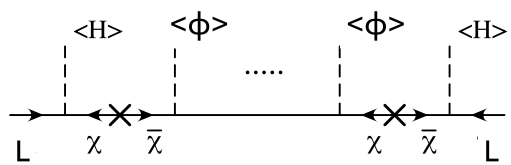
M.-C.C, de Gouvea, Dobrescu
Phys. Rev. D75, 055009 (2007)

- SM × $U(1)_{NA}$ + N ν_R
- $U(1)_{NA}$ broken by <ϕ>
- forbid usual dim-4 (Dirac) & dim-5 (HHLL) operators for neutrino mass

$$m_{LL} \sim \frac{HHLL}{M} \rightarrow M \sim 10^{14}\ GeV$$

- neutrino masses generated by operators with high dimensionality

$$m_{LL} \sim \left(\frac{\langle\phi\rangle}{M}\right)^p \frac{HHLL}{M} \rightarrow M \sim TeV, \quad \text{for large } p \qquad \frac{\langle\phi\rangle}{M} \sim \text{not too small}$$



- charges of different fermions related through anomaly cancellation conditions ⇒ predict flavor mixing

$[SU(3)_c]^2 U(1)_\nu$	$U(1)_Y [U(1)_\nu]^2$
$[SU(2)_L]^2 U(1)_\nu$	$U(1)_\nu$ gauge-gravitational anomaly
$[U(1)_Y]^2 U(1)_\nu$	$[U(1)_\nu]^3$ ⇒ cubic equation

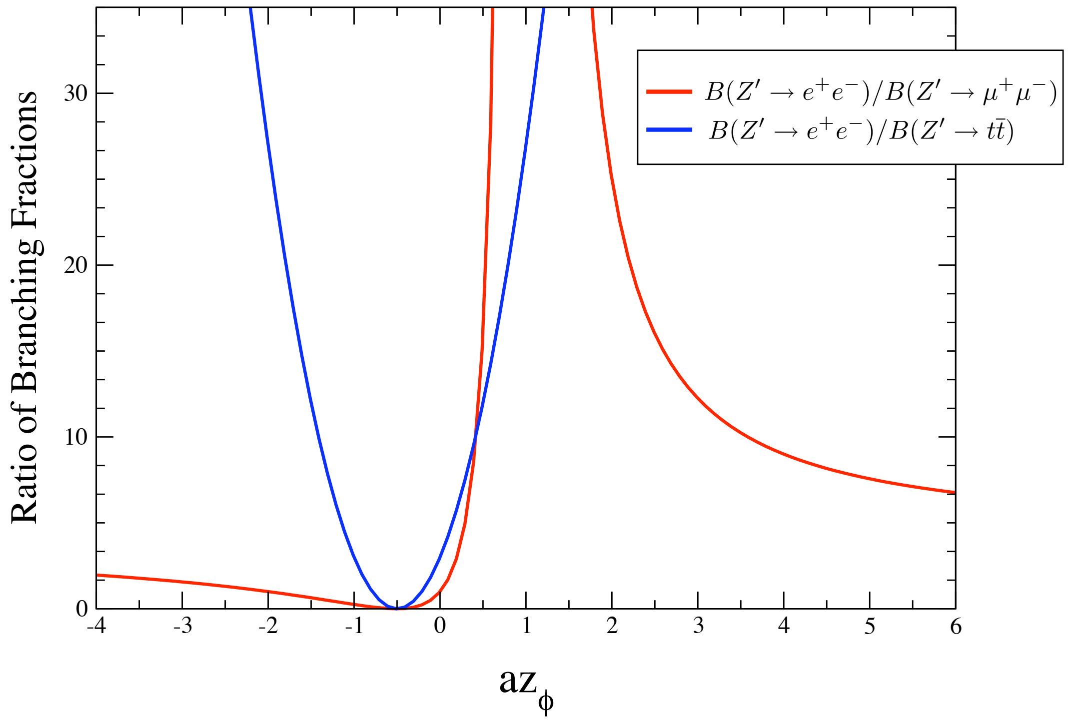
“Leptocratic” Model

- with N=3 RH neutrinos, rational solutions for charges
- UV cutoff is at ~ a TeV
- quark charges flavor blind; lepton charges flavor dependent
- neutrinos can either be Dirac (only Dirac masses are allowed) or Majorana (Dirac, LH Majorana and RH Majorana masses are all allowed)
- bi-large mixing pattern can arise
- exist light quasi-sterile neutrinos, might be relevant for cosmology

TeV Scale Seesaw with U(1)$_{NA}$

- through couplings to Z': can probe neutrino sector at colliders

  generation dependent lepton charges



Other Attempts

- SM + D^0 (vectorial quark) + S (singlet scalar) Branco, Parada, Rebelo (2003)

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}, \qquad \langle S \rangle = \frac{V \exp(i\alpha)}{\sqrt{2}}$$

$$(f_q S + f_q{}' S^*) \overline{D_L^0} d_R^0 + \tilde{M} \overline{D_L^0} D_R^0$$ →quark CPV

$$\frac{1}{2} \nu_R^{0T} C (f_\nu S + f_\nu{}' S^*) \nu_R^0$$ →leptonic CPV

- SCPV in SO(10): Achiman (2004)

<126> complex: break (B-L) $\overline{\Delta} =< \overline{\Sigma}(1,1,0) > = \frac{\sigma}{\sqrt{2}} e^{i\alpha}$

$$Y_\ell^{ij} \nu_R^i \overline{\Delta} \nu_R^j$$

- no symmetry reason why <S> is the only complex VEV

Quantum Boltzmann Equations

Buchmuller, Fredenhagen, 2000; Simone, Riotto 2007; Lindner, Muller 2007

- Classical vs Quantum Boltzmann equations:
  - collision terms: involving quantum interference
  - time evolution: quantum mechanical treatment
- Classical Boltzmann equations:

  scattering independent from previous one

$$\frac{\partial n_{N_1}}{\partial t} = -\langle \Gamma_{N_1} \rangle \left(n_{N_1} - n_{N_1}^{\rm eq} \right) ,$$

$$\langle \Gamma_{N_1} \rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{\rm eq}}{n_{N_1}^{\rm eq}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathcal{M}(N_1 \to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} (2\pi)\delta\left(\omega_{N_1} - \omega_\ell - \omega_H\right)$$

Schwinger, 1961; Mahanthappa, 1962; Bakshi, Mahanthappa, 1963; Keldysh, 1965

- Quantum Boltzmann equations:
  - Closed-Time-Path (CTP) formulation for non-equilibrium QFT
  - involve time integration for scattering terms

➡ "memory effects": time-dependent CP asymmetry

$$\frac{\partial n_{N_1}}{\partial t} = -\langle \Gamma_{N_1}(t) \rangle n_{N_1} + \langle \widetilde{\Gamma}_{N_1}(t) \rangle n_{N_1}^{\rm eq},$$

$$\langle \Gamma_{N_1}(t) \rangle = \int_0^t dt_z \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{\rm eq}}{n_{N_1}^{\rm eq}} \Gamma_{N_1}(t),$$

$$\Gamma_{N_1}(t) = 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathcal{M}(N_1 \to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} \cos\left[\left(\omega_{N_1} - \omega_\ell - \omega_H \right) (t - t_z) \right]$$

Quantum Boltzmann Equations

- time scale of Kernel $\ll$ relaxation time scale $\sim 1/\Gamma_{N1}$

  Classical Boltzmann eqs ≈ Quantum Boltzmann eqs

- In resonant leptogenesis: $\Delta M = (M_2\text{-}M_1) \sim \Gamma_{N2}$

  Kernel time scale $\sim 1/\Delta M > 1/\Gamma_{N1}$ possible

  $\Rightarrow$ quantum Boltzmann eqs important!!

Conclusions

- in minimal LR model: 2 intrinsic phases to account for all CPV in Nature
- LR parity: LH & RH Majorana mass terms proportional
- pronounced correlation between leptogenesis and low energy leptonic CPV, even without flavor effects
- CPV in quark sector may also be connected to CPV in lepton sector, if seesaw scale is low; such low seesaw scale can be obtained with an additional U(1) symmetry
- Quantum Boltzmann equations?