The Baryon Asymmetry

Experiments show that:

\[ \eta_B \equiv \frac{n_B}{s} = 9.2^{+0.6}_{-0.4} \times 10^{-11} \]

This number is roughly consistent as determined by the anti-protons in cosmic rays, relic abundance of baryonic matter, nucleosynthesis, and the CMB (which is currently the most precise determination).

The SM contained the right ingredients to explain it, but fails because the EW phase transition is predicted to be second order, and the CKM phase is not sufficiently large.
An Earlier Phase Transition?

I would like to explore the idea that some new physics operating at slightly above the EW scale is responsible for baryogenesis.

The particular idea I will explore is that the EW gauge interactions are extended to more symmetries. When these break down to the ordinary EW interactions (through a scaled up version of the Higgs mechanism), the phase transition generates B and L.

The challenge is that below the new phase transition, the ordinary EW sphalerons are still going full strength. They will try to wipe out any B I generate this way.
Top-flavor

Top-flavor expands the weak interactions into an SU(2) for the third generation, and one for the first and second generations. So we have a pair of W’s and a Z’.

The ordinary weak interactions are the diagonal subgroup (and are close to family universal).

Dimensional deconstruction suggests this has similar physics to an extra-dimensional theory of flavor.

Chivukula, Simmons, Terning PRD53, 5258 (1996)
Muller, Nandi PLB383, 345 (1996)
Malkawi, Tait, Yuan PLB385, 304 (1996)

Top-flavor

Each SU(2) has two charged and one neutral gauge boson associated with its generators.

\[ W_1^\pm, W_1^0, \quad W_2^\pm, W_2^0 \]

When the Higgs \( \Sigma \) gets an expectation value (u), it breaks SU(2)\( \times \)SU(2) down to a “diagonal” SU(2) whose generators are a linear combination of the original generators.

\[
\begin{bmatrix}
W \\
W'
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
W_1 \\
W_2
\end{bmatrix}
\]

\[ g_1 = \frac{g}{\sin \phi} \quad g_2 = \frac{g}{\cos \phi} \]

This residual symmetry is identified with our usual Electroweak force (Ws and Z). The broken combinations become massive and are new states (W' and Z'), not found in the Standard Model.

\[ M^2_{W', Z'} = \frac{g^2}{2 \sin^2 \phi \cos^2 \phi} u^2 \]
Interactions

- At extremely high energies, well above the W’ and Z’ masses, we can think in the original SU(2)_1–SU(2)_2 basis.

- In that limit, we see a different W and Z for the third family quarks and leptons from the W and Z that couples only to the first two families.

- At lower energies, I should think about the mass eigenstates (W and W’ for example).

- The W (and Z) couple (approximately) universally to all of the fermions, regardless of family.

- The W’ and Z’, instead, reflect the fact that the third family is different, and have couplings which depend on the family.

\[
W' - t - b : (g \cot \phi) \gamma^\mu P_L \quad \text{W' - u - d : (g \tan \phi) \gamma^\mu P_L}
\]

enhanced \quad suppressed
Is Topflavor Useful!?

- Top-flavor has had many incarnations:
  - Non-commuting Extended Technicolor, to explain the large top mass in a technicolor context.
  - To help with some peculiarities of the precision EW data (originally $R_b$, perhaps now $A_{FB}^b$).
  - As a (deconstruction of an) extra dimensional model to explain hierarchies in the Yukawa interactions.
  - In supersymmetric theories, to raise the Higgs mass (at tree level) above the LEP II limit to reduce fine tuning.

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Chivukula, Simmons, Terning PRD53, 5258 (1996)

Malkawi, Tait, Yuan PLB385, 304 (1996)


Batra, Delgado, Kaplan, Tait JHEP0402, 043 (2004)
Top-flavor Breaking

The breakdown of SU(2) x SU(2) -> SU(2) is through the Higgs $\Sigma$ which is a (2,2).

Under the ordinary EW symmetry, it decomposes into a singlet and a triplet:

$$\Sigma = \frac{1}{2} \begin{pmatrix} \sigma + \tau_3 & \sqrt{2}\tau_+ \\ \sqrt{2}\tau_- & \sigma - \tau_3 \end{pmatrix}$$

The breakdown is due to a VEV in the singlet component, $\sigma$, induced by a potential:

$$V_\Sigma = m^2|\Sigma|^2 + \lambda |(\Sigma\Sigma)|^2 + \lambda' |\Sigma|^4 + (-\frac{1}{2} D(\Sigma\Sigma) + \tilde{\lambda}(\Sigma\Sigma)|\Sigma|^2 + h.c.)$$

The VEV $u$ is generally complex:

$$\langle \Sigma \rangle = \begin{pmatrix} u_0 e^{i\theta_0} & 0 \\ 0 & u_0 e^{i\theta_0} \end{pmatrix}$$

$$\theta_0 = \frac{1}{4} \text{acos} \text{ Re} \begin{bmatrix} -2D^* + \tilde{\lambda}^* u_0^2 \\ -2D + \lambda u_0^2 \end{bmatrix}$$
Fermion Masses

We can generate third family Yukawa interactions very easily because the Higgs is charged under the same SU(2) as the third family doublets.

To generate the first two family fermion masses, we include a “spectator” Higgs $H'$.

The $\Sigma$ Higgs acts as a bridge between $H$ and $H'$, giving mass to the light fermions, i.e.:

$$A_1 H' \Sigma H^\dagger + h.c$$

These interactions will turn out to be important later!
Constraints

- Precision EW constraints were considered by a number of authors.

- The most stringent bounds come from non-universality of the third family couplings to the $Z$ (bottom and $\tau$). We do a global fit to LEP and SLD.

- Instantons also bound the coupling from proton decay.

\[ M_{W',Z'}^2 = \frac{g^2}{2 \sin^2 \phi \cos^2 \phi} u^2 \]

D. Morrissey, TT, C. Wagner. PRD72:095003,2005

Allowed by $p$ decay
Phase Transition

At high energies, top-flavor has two sectors of instantons, one acting on the third family and one on the first two families.

I’ll consider the limit of large coupling in the first SU(2), so I can neglect the first two families.

The phase transition will produce third family quarks and leptons (Δ of each, with B−L=0). The baryons will quickly diffuse into all three families, because of the large quark masses and CKM angles.

The third family lepton number is frozen in the tau and its neutrino because ν masses are so small.
Top-Instantons

- Each SU(2) gauge theory has its own sector of Sphalerons.
- The first SU(2) creates 3 third family quarks and one third family lepton.
- The second SU(2) creates 3 first and 3 second family quarks and one each of a first and second generation lepton.
- When $g_1 \gg g_2$, I can neglect the second process (it happens too slowly compared with the first one).
- Once the SU(2)’s break down into the ordinary weak interactions, we have the ordinary Electroweak sphalerons, each one creating 9 quarks and 3 leptons, divided evenly among the families.
Below the top-flavor phase transition scale, the EW sphalerons are still active.

Since B-L=0, they can set B=L=0. So they DO erase the baryon asymmetry we have generated.

But what they can’t do is change the distribution of lepton number inside each family individually.

So we end up with:

\[ B = 0; \quad L_1 = L_2 = -\frac{\Delta}{3}; \quad L_3 = \frac{2\Delta}{3}\]

It turns out this will be good enough!
EW Phase Transition

- The Universe persists with no net baryon number until we reach the ordinary EW phase transition.

- At that point, masses for the fermions turn on.

- We can write the number densities of the fermions in terms of chemical potentials (in the limit $T \gg m$):

  $L_i \approx \frac{1}{2} \mu_i T^2 \beta_i$

  $\beta_i \equiv 1 - \frac{1}{\pi^2} \frac{m_i^2}{T^2}$

  $B \approx -\frac{1}{3} \mu T^2 \alpha$

  $\alpha \equiv 6 - \frac{3}{2\pi^2} \sum_{i=1}^{6} \frac{m_{q_i}^2}{T^2}$

- EW sphalerons (+ fast flavor changing weak interactions) conserve three quantities:

  $\Delta_i \equiv L_i - \frac{1}{3} B \approx \frac{\mu T^2}{9} \alpha - \frac{\mu_i T^2}{2} \beta_i$
...Resulting in $B \neq 0$!

- So for any system that starts with three $\Delta$ with some values, we can compute the resulting baryon density by inverting the three equations for $\mu$:

  $$\Delta_i \equiv L_i - \frac{1}{3} B \approx \frac{\mu T^2}{9} \alpha - \frac{\mu_i T^2}{2} \beta_i$$

- Keeping the leading terms (assuming $B-L=0$), we obtain:

  $$B = - \frac{4}{13\pi^2} \sum_{i=1}^{N} \Delta_i \frac{m_{l_i}^2}{T^2}$$

- Remarkably, even for $B-L=0$, as long as not all of the $\Delta$'s are zero, we do end up with a non-zero $B$. Putting in the $\tau$ lepton mass and $T \sim 100$ GeV, we find the final baryon asymmetry is of order: $10^{-6} \Delta_\tau$
Top-flavor Baryogenesis

So in order to generate the correct baryon asymmetry of the Universe, Top-flavor must:

- Have a strongly first order phase transition so we will move to the broken phase through bubble nucleation, with the top-flavor sphalerons switching off sharply as one crosses the bubble wall.

- Generate a $\tau$-lepton asymmetry of about $10^{-4}$.

- Do both of these for parameters not in conflict with any existing measurements.
Phase Transition

Making the phase transition first order is relatively simple.

It is well known that gauged Higgs theories generally have first order phase transitions for strong enough gauge couplings.

As in the SM, we need the quartic for the Higgs (in this case $\Sigma$) to be small.

We choose:

$$\lambda = \lambda' = \tilde{\lambda} = 0.05$$

$$D = 5 \times 10^5 e^2 \text{GeV}^2$$

$$\sin^2 \phi = 0.4$$

Leading to zero Temperature results:

$$u_0 \simeq 2.7 \text{ TeV}$$

$$\theta_0 \simeq -0.7$$

$$M_{Z'} \simeq 2 \text{ TeV}$$

$$V_{\Sigma} = m^2 |\Sigma|^2 + \lambda |(\Sigma \Sigma)|^2 + \lambda' |\Sigma|^4 + \left( -\frac{1}{2} D (\Sigma \Sigma) + \tilde{\lambda} (\Sigma \Sigma)|\Sigma|^2 + h.c. \right)$$

*: Consistent with EW precision and proton decay...
We choose tree level parameters and can compute the corrections to the $\Sigma$ potential.

The important corrections come from the $W$'s and $Z'$. (Self-interaction corrects are small because the $\lambda$'s are).

$$V(u, \theta, T) = V(u, \theta, 0) + V_1(u, \theta, 0) + V_1(u, \theta, T)$$

$$V_1(u, \theta, 0) = \frac{6}{64\pi^2} \left( \frac{g_L^2}{s^2 c^2} \right)^2 u^2 \left[ u^2 \left( \log \frac{u^2}{u_0^2} - \frac{3}{2} \right) + 2u_0^2 \right]$$

$$V_1(u, \theta, T) = \frac{g_i T^4}{2\pi^2} \int_0^\infty dx \cdot x^2 \left\{ \log \left( 1 - \exp^{-\sqrt{x^2 + g^2 u^2 / (s^2 c^2 T^2)}} \right) \right\}$$
**Bubble Wall Profile**

To study the parameters of the nucleated bubble, we can use the potential at the critical temperature.

We use an ansatz (based on a 3d kink) for the VEV as a function of radius:

\[
u(r) = \frac{\nu_c}{2} [1 - \tanh (\alpha (r - R))] \\
\theta(r) = \theta_{u=0} + \frac{\theta_c - \theta_{u=0}}{2} [1 - \tanh (\alpha (r - R))]
\]

We determine \( \alpha \) variationally, by inserting the ansatz in the action at \( T_c \) and minimizing with respect to \( \alpha \).

\[
S_3(T) = 4\pi \int dr \ r^2 \left\{ \left( \nabla \langle \sigma \rangle \right)^2 + V(\langle \sigma \rangle, T_c) \right\}
\]

\[
L_w \sim \frac{10}{T}
\]
Equilibrium Relations

The net particle number for each species can be written in terms of a chemical potential $\mu$:

$$n_i = k_i \mu_i \frac{T^2}{6}$$

The $k$'s account for the internal degrees of freedom.

$$k_Q = 6; \quad k_L = 2; \quad k_t = k_b = 3; \quad k_h = 8$$

Fast weak/Yukawa interactions and strong instantons allow us to relate all of the light quark densities in terms of the right-handed bottom quark:

$$Q_{1L} = Q_{2L} = -2U_R = -2D_R = -2S_R = -2C_R = -2b$$
Which brings us to the diffusion equations:

\[ \nu_w Q' - D_Q Q'' = -\Gamma_y \left[ \frac{Q}{k_Q} - \frac{h}{k_h} - \frac{t}{k_t} \right] - 6\Gamma_{QCD} \left[ 2\frac{Q}{k_Q} - \frac{t}{k_t} - 9\frac{b}{k_b} \right] \]

\[ v_w t' - D_t t'' = -\Gamma_y \left[ -\frac{Q}{k_Q} + \frac{h}{k_h} + \frac{t}{k_t} \right] + 3\Gamma_{QCD} \left[ 2\frac{Q}{k_Q} - \frac{t}{k_t} - 9\frac{b}{k_b} \right] + \gamma_h \]

\[ \nu_w h' - D_h h'' = -\Gamma_y \left[ -\frac{Q}{k_Q} + \frac{h}{k_h} + \frac{t}{k_t} \right] + \gamma_h \]

\[ \nu_w b' - D_b b'' = 3\Gamma_{QCD} \left[ 2\frac{Q}{k_Q} - \frac{t}{k_t} - 9\frac{b}{k_b} \right] \]

\[ \nu_w L' - D_L L'' = -2\Gamma_1 \left[ 3\frac{Q}{k_Q} + \frac{L}{k_L} \right] \]

\( \Gamma_1 \): Top–flavor sphalerons
\( \Gamma_{QCD} \): strong instantons
\( \Gamma_y \): Top Yukawa interaction

CP-violating source
The CP-violating source is induced by the bubble wall.

The change in the VEV of $\Sigma$ is accompanied by a change in the CP-violating phase.

$\Sigma$ interacts with the Higgs through terms such as:

$$\langle \Sigma(x) \rangle \quad \sim \left( \frac{\Delta \theta}{L_w} v_w \right) u^2(x) |A_1|^2 I \quad \sim 10^8 \text{ GeV}^4$$

This induces a shift in the properties of the Higgs inside and outside of the bubble, resulting in an imbalance in the Higgses and anti-Higgses transmitted/reflected from the bubble wall. The source is non-zero only in the wall.
Quasi-analytic Solution

The rates of the processes $\Gamma_1$, $\Gamma_y$, and $\Gamma_{QCD}$ are all comparable to the typical diffusion rates:

$$\Gamma_y \simeq \frac{27}{2} \lambda_t^2 \alpha_s \left( \frac{\zeta(3)}{\pi^2} \right)^2 T = 7.4 \text{ GeV}$$

$$\Gamma_{QCD} \simeq 16\kappa' \alpha_s^4 T = 0.3 \text{ GeV}$$

$$\Gamma_1 \simeq 30\alpha_1^5 T = 0.1 \text{ GeV}$$

$$D_{L}^{-1} \sim T/110 \sim \text{ GeV}$$

This means these processes are approximately in equilibrium, and we can impose the equilibrium conditions on the diffusion equations to simplify their solution.

Simpler Equations

To proceed, we take linear combinations of the diffusion equations which are independent of the three rates $\Gamma$. We then replace $L$, $t$, and $b$ with their equilibrium values:

\[
2 \frac{Q}{k_Q} - \frac{t}{k_t} - 9 \frac{b}{k_b} \sim \frac{1}{\Gamma_{QCD}} \sim 0
\]

\[
\frac{Q}{k_Q} - \frac{h}{k_h} - \frac{t}{k_t} \sim \frac{1}{\Gamma_y} \sim 0
\]

\[
3 \frac{Q}{k_Q} + \frac{L}{k_L} \sim \frac{1}{\Gamma_1} \sim 0
\]

(on the unbroken side)
Simplified Diffusion

... to get a pair of coupled equations for Q and h:

\[
M \begin{bmatrix} Q'' \\ h'' \end{bmatrix} + N \begin{bmatrix} Q' \\ h' \end{bmatrix} = \begin{bmatrix} -\gamma h \\ 0 \end{bmatrix}
\]

\[
M = \begin{bmatrix}
D_Q \left( \frac{k_b-9k_t}{9k_b} \right) & D_h + D_Q \left( \frac{9k_t+k_b}{9k_h} \right) \\
-D_Q \left( \frac{9k_Q+9k_t+k_b}{9k_Q} \right) - D_L \left( \frac{9k_L}{k_Q} \right) & D_Q \left( \frac{9k_t-k_b}{9k_h} \right)
\end{bmatrix} = \frac{1}{T} \begin{bmatrix} -\frac{8}{3} & \frac{225}{2} \\
-\frac{28}{3} & 2 \end{bmatrix}
\]

\[
N = v_w \begin{bmatrix}
\frac{9k_t-k_b}{9k_Q} & -\frac{9k_h+9k_t+k_b}{9k_h} \\
\frac{9k_Q+81k_L+9k_t+k_b}{9k_Q} & \frac{k_b-9k_t}{9k_h}
\end{bmatrix} = v_w \begin{bmatrix}
\frac{4}{9} & -\frac{17}{3} \\
\frac{41}{9} & \frac{12}{3}
\end{bmatrix}
\]

We solve these equations separately in the region inside and outside the bubble wall (broken, and unbroken phases, solutions of the homogeneous equations) and inside the wall (where the source is on).
Success!

- Solving these equations yields the profile of the particle densities.
- We place the wall at \( z=0 \). Its width is \( \sim 10/T \sim 10^{-2} \text{ GeV}^{-1} \).
- We find for the parameters we have chosen, we arrive at \( \Delta_T \) of order \( 10^{-4} \), which including the dilution of \( 10^{-6} \) yields about the right \( B \).

(We have a numerical solution as well; it is pretty much identical)
Outlook

The specific result is (of course) dependent on the choice of parameters. The value itself is not very interesting.

What is interesting is that for “natural” values of parameters, one can obtain the right ballpark for the baryon density.

Further, there are systematic features which decide whether or not this works in the Topflavor model:

- A small quartic for Σ implies a light mass for the radial mode: about a factor of 3 lighter than the W'/Z' masses.
- Large CP violation which may be visible in Σ decays (which are mostly to Higgs).
Outlook

- We can search for the W’s and Z’s at the LHC.
- For example, the W’s lead to an enhancement of s-channel single top production!
It is interesting that the idea of baryogenesis through a phase transition may not have to be abandoned, even if the EW phase transition turns out to have been second order.

At any rate, low scale models are generally testable at the LHC, and so we don’t have long to find out something!

Other gauge-extended models, such as the ununified model, can lead to similar phenomenon!

Georgi, Jenkins, Simmons, PRL62, 2789 (1989)
Supplemental Slides
Big Bang Nucleosynthesis

The ratios of the primordial elements is also sensitive to the baryon asymmetry.

The Li abundances are marginally inconsistent with the most accurate determinations from deuterium and Helium.

Y is more or less equivalent to the fraction of $^4\text{He}$. 
The relative sizes of the doppler peaks of the CMB are sensitive to the number of baryons in the Universe.

Uses for Topflavor?

- Top-flavor has had many incarnations:
  - Non-commuting Extended Technicolor, to explain the large top mass in a technicolor context.
  - To help with some peculiarities of the precision EW data (originally $R_b$, perhaps now $A_{FB}^b$).
  - As a (deconstruction of an) extra dimensional model to explain hierarchies in the Yukawa interactions.
  - In supersymmetric theories, to raise the Higgs mass (at tree level) above the LEP II limit to reduce fine tuning.

Batra, Delgado, Kaplan, Tait JHEP0402, 043 (2004)
Diffusion Equations

Now we determine and solve the differential equations which describe the particle number densities induced by the passage of the wall.

The diffusion equations are based on the fact that an imbalance in number densities participating in any interaction which is taking place quickly will look like a source or sink for those species which are imbalanced.

Processes which are extremely fast will maintain chemical equilibrium, and allow us to reduce the number of species we include the equations. (Other species are related to those by equilibrium conditions).

Diffusion Equations

So the species we will consider are: $Q_3, t_R, b_R, H, L_3$

The current conservation equation can be written:

$$\partial_\mu J^\mu \simeq \partial_0 n - D \vec{\partial}^2 n \simeq v_w n' - D n''$$

where the diffusion constants $D$ depend on the rate of interaction with the background plasma:

$$D_Q \sim D_t \sim D_b \sim 6/T \quad \text{and} \quad D_h \sim D_L \sim 110/T$$

and $v_w$ is the velocity of the bubble wall as it expands, typically 0.01–0.1 for weakly coupled theories.